

A New Version of Reimers' law of Mass Loss Based on a Physical Approach

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ABSTRACT

We present a new semi-empirical relation for the mass loss of cool stellar winds, which so far has frequently been described by “Reimers’ law”. Originally, this relation was based solely on dimensional scaling arguments without any physical interpretation. In our approach, the wind is assumed to result from the spill-over of the extended chromosphere, possibly associated with the action of waves, especially Alfvén waves, which are used as guidance in the derivation of the new formula. We obtain a relation akin to the original Reimers law, but which includes two new factors. They reflect how the chromospheric height depends on gravity and how the mechanical energy flux depends, mainly, on effective temperature. The new relation is tested and sensitively calibrated by modelling the blue end of the Horizontal Branch of globular clusters. The most significant difference from mass loss rates predicted by the Reimers relation is an increase by up to a factor of 3 for luminous late-type (super-)giants, in good agreement with observations.

Subject headings: stars: chromospheres — stars: late-type — stars: mass loss — turbulence — waves

1. Introduction

Empirical mass loss formulas are pivotal for the construction of empirical and semiempirical stellar atmosphere and wind models, stellar evolution computations and studies of the interstellar medium, among other topics. Historically, the mass loss rate \dot{M} of late-type giants and supergiants has been described by “Reimers’ law”, given as $\dot{M} = \eta \cdot \frac{L_* R_*}{M_*}$ (Reimers 1975, 1977), with L_* , R_* , M_* as stellar luminosity, radius, and mass, respectively, given in solar units, and η is a fitting parameter. Other empirical mass loss formulas have been presented by Lamers (1981), de Jager, Nieuwenhuijzen, & van der Hucht (1988), and Nieuwenhuijzen & de Jager (1990), but they do not distinguish between the strong, and now well-described dust-driven winds (e.g., Wachter et al. 2002), and the physically very different case of nondust-driven winds.

Despite its wide-ranging success, the mass loss formula by Reimers suffers from two important deficiencies. First, it is solely based on dimensional scaling arguments without any physical interpretation. In particular, the appearance of the stellar luminosity in the formula is awkward noting that for cool star winds, with the exception of molecule-driven and dust-driven winds, the luminosity of the star is not expected to be relevant (e.g., Holzer & MacGregor 1985). In fact, the Reimers law seems to suggest that a certain fraction of the stellar luminosity L_* is utilized to lift the wind material from the photosphere. The second deficiency consists in the necessity of adjusting the fitting parameter η , as different η values are required ad hoc to match observed mass-loss rates from different types of giants and supergiants. The same is true, if reasonable mass loss yields and final masses are to be achieved through stellar evolution models with prescribed mass loss. For more evolved AGB giants, the Reimers relation is better replaced by, e.g., de Jager et al. (1988), which suggests up to three times as much mass-loss for the tip-AGB (Schröder & Sedlmayr 2001). More recently, the Reimers relation also fails to describe revised mass loss rates from K and M giant stars, based on updated Ca II ionization balances which consider photoionization radiation deduced from FUSE spectra (Harper et al. 2004). For further updated information on mass-loss mechanisms see, e.g., the review by Willson (2000).

In the present work, we overcome these deficiencies by adopting a more physical picture. In our approach, the non-radiative energy input into the wind is assumed to be given by the turbulent energy density, within the chromosphere or underneath, possibly related to the manifestation of (magneto-)acoustic waves. This approach appears to be consistent with the major conclusion by Judge & Stencel (1991), who presented a detailed empirical analysis of the global thermodynamical properties of the outer atmospheres and winds of a set of well-studied cool giant and supergiant stars. They concluded that “[...] mass loss rates are not strongly dependent on the actual physical processes driving the winds [suggesting] that

nonlinear processes act to regulate wind energy fluxes”. Furthermore, we assume that the mass loss rate depends on the characteristic chromospheric height, which dictates the amount of energy required to lift the wind out of the potential well of the star. This simplified model results in a mass loss formula akin to that by Reimers (1975, 1977). However, it contains two additional factors, one depending on the effective temperature and the second on the surface gravity of the star. This improves the agreement with observed mass loss rates for different types of stars, without the need to adjust the fitting parameter. In §2, we describe our theoretical approach. In §3, we discuss tests and applications of our new formula, and in §4, we give our conclusions.

2. Theoretical Approach

In our simplified model, we regard the wind to result from a spill-over of the extended, highly turbulent giant chromosphere and its reservoir of mechanical energy, possibly associated with waves. Even though no details of any theoretical models are considered, the following derivation will be guided by the assumption of Alfvén waves, owing to their success in describing stellar winds as obtained for α Boo (K1.5 III) (Hartmann & MacGregor 1980), ζ Aur A (K4 Ib) (Kuin & Ahmad 1989), and α Ori (M2 Iab) (Hartmann & Avrett 1984; Airapetian et al. 2000). The relevant mechanical energy flux F_M may thus be due to magnetic energy generation, a consequence of non-isotropic chromospheric turbulence, or a combination of both.

Turbulence is a well-known feature of stellar photospheres (e.g., Gray 1992, and references therein, among more recent literature), and of cool star chromospheres. E.g., Carpenter (1996) obtained empirical constraints on the chromospheric macroturbulence and flow velocities for various K and M (super-)giants based on C II] from HST-GHRS spectra, ranging from 24 km s^{-1} (α Tau; K5 III) to 35 km s^{-1} (α Ori; M2 Iab), which are in principle sufficient to overcome the gravitational potential of the star. Nevertheless, the chromospheric turbulent energy density relevant for the generation of winds is not exactly known, largely because of the difficulty of distinguishing between isotropic and non-isotropic turbulence. Examples of wave-driven wind models have also been given. For instance, Airapetian et al. (2000) proposed a time-dependent, 2.5-D Alfvén wave wind model, resulting in a time-averaged mass loss rate commensurate with the recent semi-empirical chromosphere and wind model by Harper, Brown, & Lim (2001) based on NRAO VLA radio data.

Aside from the amount of utilized mechanical energy flux, the stellar mass loss rate is also expected to depend on the characteristic chromospheric radius R_{Chr} , which dictates the amount of wind energy dE_{Wind} needed by a mass element $dM = \dot{M}dt$ to overcome the

gravitational potential of the star. For the wind energy balance, we thus obtain

$$dE_{\text{Wind}} \simeq \frac{GM_* \dot{M} dt}{R_{\text{Chr}}} \propto F_{\text{M}} \cdot 4\pi R_*^2 dt \quad (1)$$

with \dot{M} as mass loss rate, and R_* , M_* as stellar radius and mass, respectively, F_{M} as mechanical energy flux, and G as gravitational constant¹.

A large body of literature has been devoted to describing convective turbulence of stellar atmospheres and the generation of waves as function of the fundamental stellar parameters. Stein (1981) studied the generation of waves by turbulent motions in stellar atmospheres largely based on analytic means. He found that the acoustic wave energy flux is given as $F_{\text{M}} \propto T_{\text{eff}}^{6.1}$ (monopole term), $T_{\text{eff}}^{10.4}$ (dipole term), and $T_{\text{eff}}^{14.6}$ (quadrupole term) (see representation by Ulmschneider [1989]), noting that the monopole term is most closely and the quadrupole term is least closely related to mass loss generation. Models by Bohn (1984) deduce as temperature dependence for the monopole, dipole, and quadrupole terms, $T_{\text{eff}}^{4.15}$, $T_{\text{eff}}^{8.78}$, and $T_{\text{eff}}^{13.85}$, respectively, and for the combination of those terms $T_{\text{eff}}^{9.75}$. Analytic models for magnetic wave generation reveal a temperature dependence of $T_{\text{eff}}^{7.5}$ for the Alfvén mode and T_{eff}^8 for the magnetic modes combined (Musielak & Rosner 1988).

More recent work has resulted in vast improvements of these models, particularly with respect to the models for the stellar convection zones and the description of the turbulent frequency spectra — see, e.g., Musielak (2004) for a recent review on those results. Unfortunately, the authors usually refrain from giving a fitting formula for the dependence of the wave energy fluxes on the stellar effective temperatures, supposedly because of the large number of free parameters, particularly in magnetic models. Nevertheless, based on the fact that the solar wind (e.g., Ong et al. 1997) and massive stellar winds (e.g., Rosner et al. 1995; Airapetian et al. 2000) are likely to be accelerated by the momentum deposition by Alfvén waves, we consider a dependence as $T_{\text{eff}}^{7.5}$ to be most representative. There is also another feature inherent to Alfvén waves motivating this choice: they are known to be essentially non-dissipative in the lower and middle chromosphere — see discussion by Carbonneau & MacGregor (1995) and Boynton & Torkelsson (1996), pointing to the prevalence of low-amplitude waves in those stars — implying that the same T_{eff} -exponent holds in the region of wave generation as well as where the onset of mass loss occurs. Incidentally, the

¹Historically, a similar approach has been undertaken by Fusi-Pecchi & Renzini (1975), who assumed that cool star winds are due to acoustic waves. Later on, this assumption has been invalidated by detailed model simulations, which showed that acoustic waves fail to support significant mass loss as they dissipate most of their energy immediately beyond the stellar photosphere, and are thus incapable of meeting the potential energy requirement of stellar winds (e.g., Cuntz 1990; Sutmann & Cuntz 1995, and references therein).

same exponent is also found as temperature dependence of Mg II h , k emission in stars (both dwarfs and giants) of minimal activity, a likely indicator of the overall chromospheric energy density (Buchholz, Ulmschneider, & Cuntz 1998, see their Fig. 15).

If the mechanical energy flux F_M utilized for generating stellar mass loss is assumed as $F_M \propto T_{\text{eff}}^{7.5}$, the surface-integrated mechanical energy flux L_M can now be expressed as

$$L_M = F_M \cdot 4\pi R_*^2 \propto F_M \cdot L_*/T_{\text{eff}}^4 \propto L_* \cdot T_{\text{eff}}^{3.5} \quad . \quad (2)$$

Next we consider the characteristic chromospheric radius R_{Chr} . For cool giants and supergiants, no well-defined boundary between the chromosphere and the wind exists. Hence, we use the sonic point of the average velocity field as reference. For the well-studied K supergiant ζ Aur (with $\log g_* \simeq 0.8$), R_{Chr} is found to be close to $2R_*$ (Baade et al. 1996), and for general giants and supergiants, $(R_{\text{Chr}} - R_*)/R_*$ is assumed to vary as g_*^{-1} , which gives

$$R_{\text{Chr}} = R_* \cdot \left(1 + \frac{g_{\odot}}{4300 \cdot g_*}\right) \quad . \quad (3)$$

With the above temperature dependence of the mechanical energy flux (eq. [2]) and chromospheric radius R_{Chr} (eq. [3]), we finally obtain as mass loss rate \dot{M} , see eq. (1),

$$\dot{M} = \eta \cdot \frac{L_* R_*}{M_*} \cdot \left(\frac{T_{\text{eff}}}{4000 \text{ K}}\right)^{3.5} \cdot \left(1 + \frac{g_{\odot}}{4300 \cdot g_*}\right) \quad (4)$$

with R_* , M_* , and L_* as stellar radius, mass, and luminosity given in solar units, and g_* and g_{\odot} as stellar and solar surface gravity, respectively. This is, apart from the two new factors, indeed the old Reimers law. To satisfy the well-constrained RGB mass-loss of globular cluster stars (see § 3), the fitting parameter η will be set to $8(\pm 1) \times 10^{-14} M_{\odot} \text{ yr}^{-1}$.

3. Tests and Applications

For the newly developed mass loss formula various tests and applications have been devised. In particular, we want to obtain insight into the importance of the new factors given by the stellar effective temperature T_{eff} and gravity g_* . In fact, for ordinary giants the two new factors do not make much difference, which explains the long-lasting success of the Reimers relation. In particular, the $T_{\text{eff}}^{3.5}$ factor is, despite its high power, restricted in its impact by the small band of relevant effective temperatures (3000 to 4500 K), and the g -sensitive factor remains the order of one for all but the smallest gravities. In fact, as previously discussed, the T_{eff} exponent in eq. (4) is somewhat uncertain. However, due to the narrow band of relevant effective temperatures, the overall results would still stand if T_{eff}^3 or T_{eff}^4 were used instead.

Nevertheless, we have obtained evidence for the extra dependence on T_{eff} from a comparative study of the RGB mass loss of globular cluster (GC) stars with very different metallicity and, accordingly, different effective temperatures on their RGBs. The mass lost on the RGB of a GC is very well constrained by modelling the stars at the blue end of the Horizontal Branch (HB), for which the HRD position is very mass sensitive. The remaining uncertainty is about 15 % in absolute terms and much better in relative terms. In fact, the long time (on a dynamic time-scale) spent on the RGB effectively evens out most of the inherent variability of these stellar winds. This is a big advantage over the kind of snapshots obtainable from observing individual winds directly. A residual star-to-star variation of $< 20\%$, on the other hand, is sufficient to explain the full spread of an HB.

As test cases, we consider two globular clusters, which are NGC 5904 and NGC 5927. NGC 5904 has a significant metal underabundance of $[\text{Fe}/\text{H}] = -1.29$ (normal for globular clusters, $Z=0.001$, see Fig. 1), whereas NGC 5927 only has a marginal underabundance of $[\text{Fe}/\text{H}] = -0.37$ (near $Z=0.01$, see Fig. 2). We use the photometric data and metallicities provided by Piotto et al. (2002) and have plotted our evolution tracks directly into their cluster CMDs. Clearly, the Reimers relation cannot reproduce both cases with the same η_{R} , while our new relation can! In particular, the extreme blue end of the Horizontal Branch (HB) of NGC 5904 (Fig. 1, *bottom panel*) demands HB stellar masses $M_{\text{HB}} \simeq 0.60M_{\odot}$, with a He-core mass of $M_{\text{c}} \simeq 0.49M_{\odot}$. With an approximate initial mass of $M_{\text{i}} \simeq 0.86M_{\odot}$, consistent with an age of about 12 billion years, this corresponds to a total RGB mass loss of $0.26 M_{\odot}$ for the individual GC stars. With our new mass loss relation with $\eta = 0.8 \times 10^{-13}$, our evolution models achieve exactly this HB mass. To get the same result with the old Reimers law, we would need a $\eta_{\text{R}} = 2.4 \times 10^{-13}$. On the other hand, the small extent of the HB of NGC 5927 (Fig. 2, *bottom panel*) demands HB stellar masses $M_{\text{HB}} \simeq 0.71M_{\odot}$, possibly slightly more, but certainly not less, with a He-core mass of $M_{\text{c}} \simeq 0.48M_{\odot}$. With $M_{\text{i}} \simeq 0.99M_{\odot}$, consistent with an age of about 11 billion yrs, this corresponds to a total RGB mass loss of $0.28 M_{\odot}$ — exactly as achieved by our evolution models with $\eta = 0.8 \times 10^{-13}$ and our new mass-loss relation. To get the same HB mass with the Reimers relation, we would need $\eta_{\text{R}} = 2.0 \times 10^{-13}$. On the other hand, a value of η_{R} of 2.4×10^{-13} can clearly be ruled out: it would produce HB stars with $M_{\text{HB}} \simeq 0.63M_{\odot}$, which are already far too blue.

The role of the new, gravity-related factor appearing in our new mass loss relation can be explored by assessing luminous low-gravity stars, which play a very important role in the overall mass loss during the stellar lifetime. A good test candidate is the well-studied star α Ori, which has some circumstellar dust but is still below its critical luminosity for possessing a truly dust-driven wind. The mass loss rate was observed as $\dot{M}_{\text{obs}} = 3.1(\pm 1.3) \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ (Harper et al. 2001), assuming a distance of $d = 131 \text{ pc}$ ($\pm 25\%$) according to the Hipparcos parallax. The corresponding luminosity is $L_{*} = 5.4 \times 10^4 L_{\odot}$, and with an angular diameter

of 56 mas, T_{eff} is given as 3140 K, for which matching evolution tracks imply a mass of $M_* = 10M_{\odot}$ ($\pm 30\%$ depending on L_*). For these parameters, our new mass loss relation (with $\eta = 0.8 \times 10^{-13}$) yields $2.2 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, while the Reimers law gives only $0.8 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ (with $\eta_{\text{R}} = 2 \times 10^{-13}$). The distance uncertainty for α Ori affects all three mass loss rates (including $\dot{M}_{\text{obs}} \propto d^2$), but not so much their ratios. For example, if d was 25% larger and $M_* \simeq 13M_{\odot}$, then $\dot{M}_{\text{obs}} = 4.8 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, our value would be $4.0 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, whereas the Reimers law would give $1.3 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$. In any case, for such low gravities ($\log g \simeq -0.35$), R_{Chr}/R_* increases significantly, resulting in an extra boost of mass loss.

4. Conclusions

We derived a new semi-empirical relation for the mass loss of cool winds, which so far has frequently been described by Reimers’ law. Physically, the Reimers relation appears to suggest a picture in which the wind material is lifted from the photosphere by using a certain fraction of the stellar luminosity — even though it is well known that, with the exception of molecule-driven and dust-driven winds, cool star winds are not related to any type of radiation pressure. This apparent contradiction has now been resolved.

The new relation is based on theoretical arguments assuming that the wind results from the turbulent energy density, within the chromosphere or underneath, possibly related to the manifestation of magnetoacoustic waves as, e.g., Alfvén waves. Furthermore, the mass loss rate is assumed to depend on the chromospheric extent, which dictates the amount of energy required to lift the wind out of the potential well of the star. A more detailed analysis shows that the new mass loss formula is not applicable to molecule-driven, dust-driven, and pulsational winds, as in those cases highly temperature-sensitive feed-back mechanisms exist, resulting in a steeper dependence of the mass loss rate on the stellar parameters, including the metallicity, which is not reflected by the new formula. Moreover, pulsational winds are more episodic in nature, whereas the new mass loss formula only describes time-averaged mass loss behavior. Also note that the new mass-loss formula is not valid for stars like the Sun, where information exists that different types of mass loss processes, resulting in slow and fast wind, operate on different horizontal and vertical scales, which is beyond the theoretical framework of this paper.

The new relationship mostly reproduces the original Reimers law, except that it includes two additional factors, which further improve the agreement with observed mass loss rates, especially for (super-)giants with very low gravity. This improved agreement can be interpreted as an indirect validation of Alfvén waves as being primarily responsible for mass loss generation in those stars. A highly sensitive calibration of the new relation’s fitting factor

η has been achieved by modelling the mass lost on the RGB by stars near the blue end of the Horizontal Branch, using two globular clusters of very different metallicity (NGC 5904, NGC 5927). Further studies, considering sets of well-studied K and M-type giant and supergiant stars, including comparisons with the various mass loss formulas from the literature, will be given in the near future.

We are grateful to Z. E. Musielak, D. Schönberner, R. C. Smith, and B. E. J. Pagel for helpful discussions. The paper also benefited from comments by three anonymous referees. This work has been supported by NSF under grant ATM-0087184 (M. C.).

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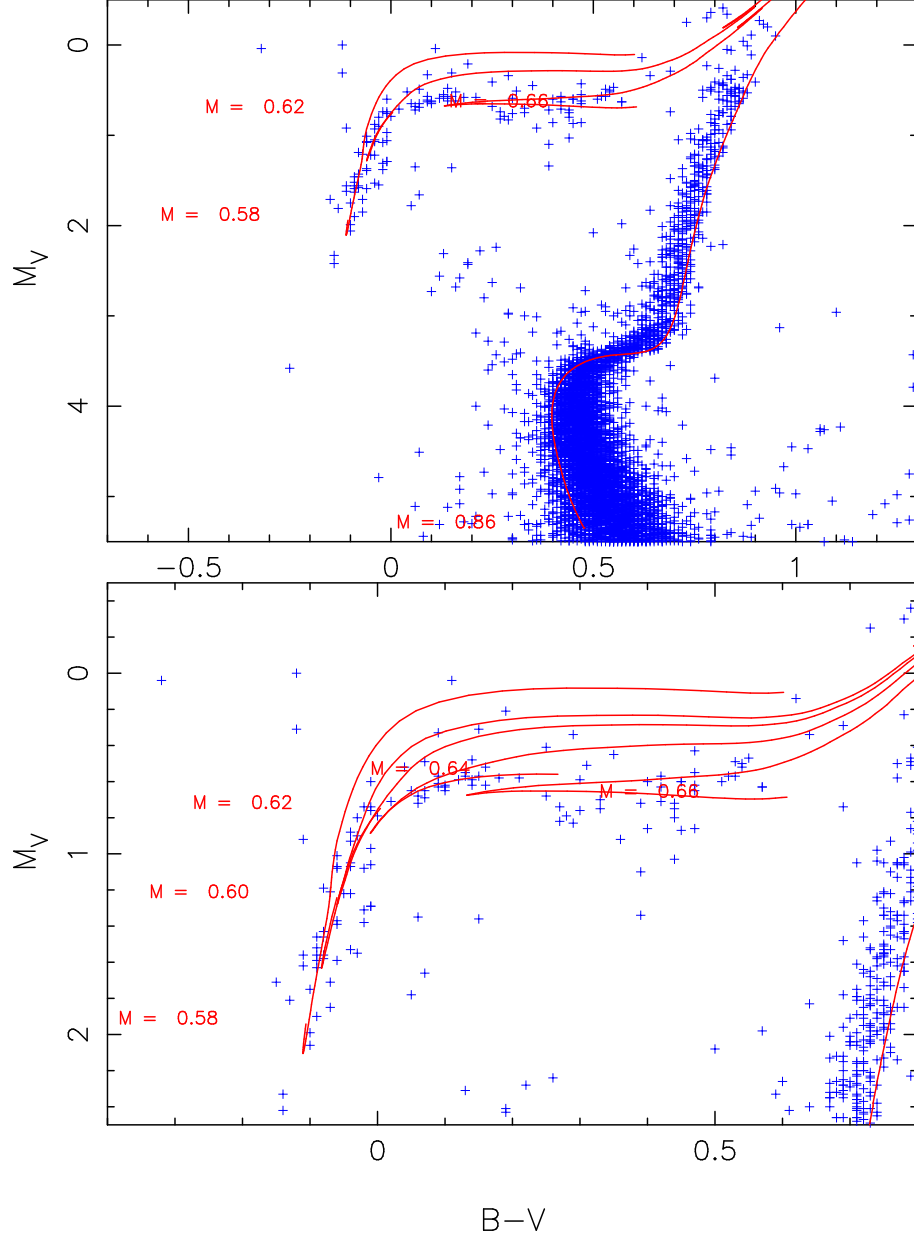


Fig. 1.— *Top panel:* Color-Magnitude Diagram of the globular cluster NGC 5904. *Bottom panel:* Position of the Horizontal Branch with $Z=0.001$.

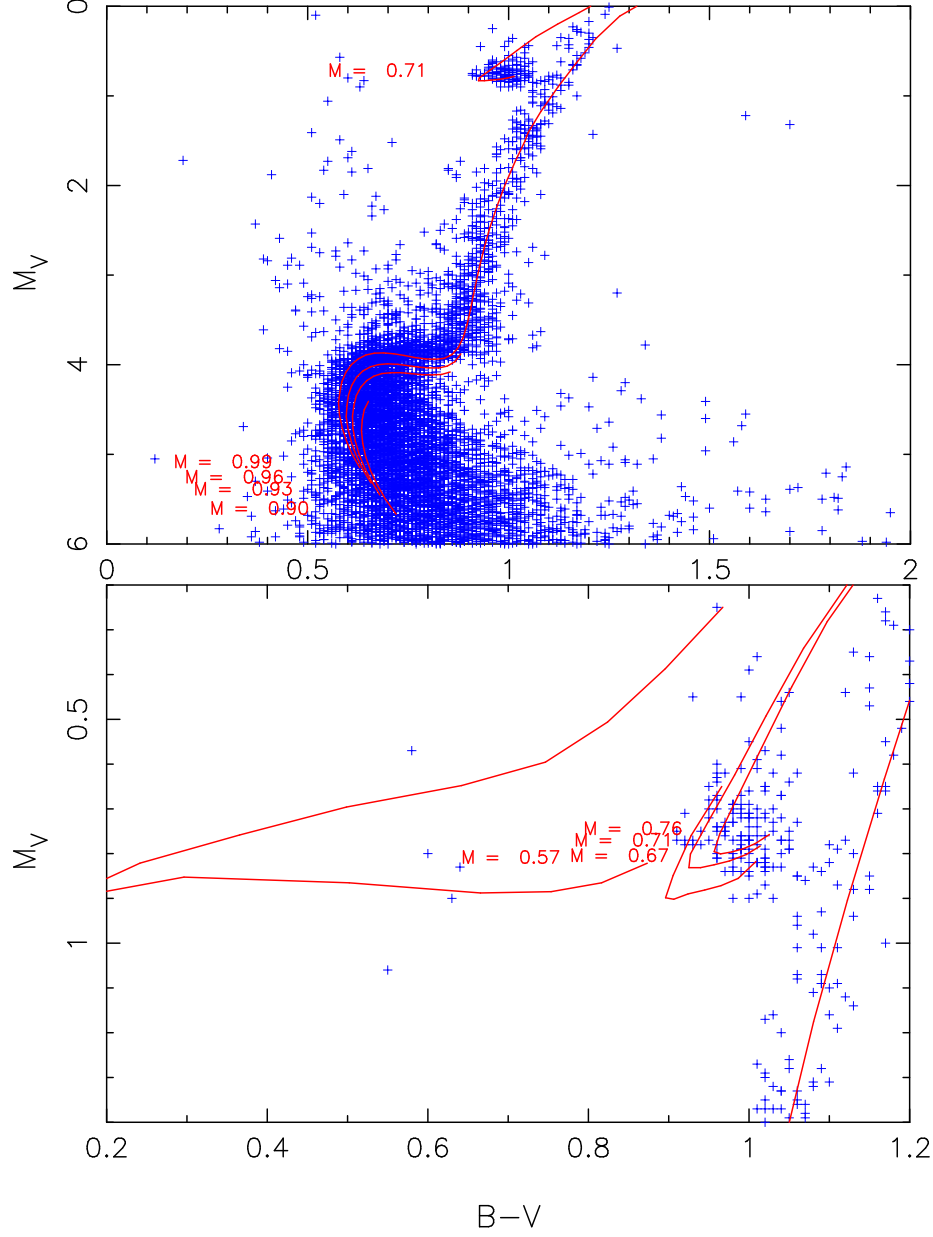


Fig. 2.— *Top panel*: Color-Magnitude Diagram of the globular cluster NGC 5927. *Bottom panel*: Position of the Horizontal Branch with $Z=0.009$.